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Some comments on the electrodynamics of binary pulsars

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ABSTRACT
We consider the electrodynamics of in-spiraling binary pulsars, showing that there are two distinct ways in which they may emit radiation. On the one hand, even if the pulsars do not rotate, we show that *in vacuo* orbital rotation generates magnetic quadrupole emission, which, in the late stages of the binary evolution becomes nearly as effective as magnetic dipole emission by a millisecond pulsar. On the other hand, we show that interactions of the two magnetic fields generate powerful induction electric fields, which cannot be screened by a suitable distribution of charges and currents like they are in isolated pulsars. We compute approximate electromotive forces for this case.

Key words: stars: neutron – pulsars: general – binaries: general

1 INTRODUCTION

The recent discovery of the binary pulsar J0737-3039A/B (Burgay et al. 2003; Lyne et al. 2004) has highlighted the fact that pulsars may retain their magnetic fields for at least a significant fraction of the time it takes them to in-spiral to their eventual merger.

This discovery has been used so far to illustrate the extraordinary opportunity to study hitherto unaccessible General Relativistic effects, while less attention has been paid to the electromagnetic interactions of the two magnetospheres (but see Kramer & Stairs 2008 also for a discussion of magnetospheric effects).

But even before this discovery, Vietri (1996) and Hansen & Lyutikov (2001) have discussed some of the consequences of assuming that the two inspiralling, merging *pulsars*, (*i.e.*, not just neutron stars) may produce observationally interesting electromagnetic signals. In particular, Hansen & Lyutikov (2001) considered the magnetospheric interaction of two inspiralling pulsars with very different magnetic fields: a fast, but weak-field pulsar, and a slow magnetar.

In this paper, we wish to take a different point of view, that in which the two fields do not differ by four to six orders of magnitude like in Hansen & Lyutikov (2001), but are more evenly matched, though not necessarily of the same order of magnitude. In particular, we wish to stress first that the ultimate energy reservoir to be tapped is the binary orbital motion, which we do by considering the simplest model possible (two point-like dipoles orbiting their common center of mass *in vacuo*), and pointing ou that this simple estimate alone leads to an observationally interesting signal. The reason is that binary pulsars will rotate ever faster as a consequence of the orbital decay induced by gravitational radiation. If the pulsars manage to retain some signifiant fraction of their magnetic fields until the moment of merging, their evolution will lead naturally to a sub-millisecond object emitting copious amounts of radiation in the last instants before merging, even when we neglect the pulsars' rotation around their spin axes. We shall first discuss this emission mechanism, which occurs *in vacuo*.

On the other hand, under these circumstances there is a natural mechanism that will create an induction electric field with a component directed along the magnetic field, for an arbitrary orientation of the axes in question. This mechanism follows from the well–known inability of the two pulsars to synchronize their spin periods with their orbital period, a direct consequence of their tiny sizes (in terms of volume, not mass, of course; see Bildsten & Cutler 1992). Since sufficiently late in the binary life the orbital period is much shorter than the spin periods, we may picture the two pulsars as non-rotating; still, the orbital motion swings each magnetosphere by the other one, so that any particular part of it is compressed on the day-side, and is free to expand on the night-side. This magnetospheric *pumping* implies a time variability of the local magnetic field, and in turn this leads to an induction electric field.

This situation differs considerably from the aligned rotator (Contopoulos et al. 1999), where a judicious choice of a stationary charge distribution inside the magnetosphere can short out the component of the electric field along the magnetic field; we shall show later that this mechanism cannot be effectively screened, as it is in the case of an isolated, rotating aligned pulsar (IRAP for short). This suggests that large-scale, large-amplitude electric fields not necessarily orthogonal to the magnetic field do exist in the magnetospheres of binary pulsars.

It is the aim of this paper to investigate some consequences of this simple idea for the (photonic) observability of in-spiraling binary pulsars as GWR pushes them closer and closer. In the next Section, we will derive the energy lost per unit time by two co-rotating magnetic moments *in vacuo*, a generalization of Pacini (1967)'s formula for the isolated magnetic moment. In Section 3,

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we consider in some detail the induction effect briefly discussed above, still using the *in vacuo* case as our model, for lack of a fully satisfactory realistic model. We shall discuss how the component of the induction electric field along a magnetic field line cannot be shorted by arbitrary charges and currents located in the magnetosphere, and we shall provide some exact and some approximate estimate of the available electromotive forces. The last Section summarizes our results.

2 IN VACUO

The starting point of this paper is a computation of the amount of energy radiated per unit time by a configuration of two point-like dipoles $\vec{\mu}_1, \vec{\mu}_2$ rotating around their common center of mass. We assume in this that the stars carrying the magnetic moments are not spinning at all ($\Omega = 0$), and call $\omega \neq 0$ the orbital angular frequency.

This configuration does not emit magnetic dipole radiation, because the total magnetic moment $\vec{\mu}_1 + \vec{\mu}_2$ is a constant, thus to this order an observer located at infinity will perceive no time-varying field. However, said observer will perceive a time–varying field to higher order, due to the fact that the magnetic field is stronger when the star with the stronger dipole moment is closer, and weaker half an orbital period later when it is furthest, and the weaker dipole is closest.

This effect depends on the orbital radius r: the modulation of the fields vanishes for $r = 0$, and it is clearly linear in r for $r \ll$ the observer's distance. Thus the radiated power will be quadratic in r, showing that we are dealing with quadrupole magnetic radiation.

It is obviously possible to obtain the radiated power by means of a vector-spherical harmonics analysis (Jackson 1975). However it is also possible to reach the same result via a simpler approach, which also allows us to introduce the fields we shall use in the next Section.

Monaghan (1968) has given exact expressions for the electromagnetic potentials and fields generated by an arbitrarily moving dipole, i.e., a particle endowed with both electric ($\vec{p}$) and magnetic ($\vec{\mu}$) dipoles. We just need to specialize to the case when, in the particle (actually a star, in our case) reference frame, it has only a non–vanishing magnetic dipole moment; in this case Monaghan shows that, when the pure dipole magnetic moment is moving with arbitrary speed $c\vec{\beta}$, it appears to have an electric dipole electric moment

$$\vec{p} = \vec{\beta} \wedge \vec{\mu} \, . \tag{1}$$

In this case the electromagnetic potentials are:

$$\vec{A} = \left(\frac{\vec{\mu} \wedge \hat{n}}{KR^2} + \frac{d}{dt}\frac{\vec{\mu} \wedge \hat{n}}{KRc} + \frac{d}{dt}\frac{\vec{\beta} \wedge \vec{\mu}}{KRc} \right)_{\rm ret} \tag{2}$$

$$\phi = \left(\frac{(\vec{\beta} \wedge \vec{\mu}) \cdot \hat{n}}{KR^2} + \frac{d}{dt}\frac{(\vec{\beta} \wedge \vec{\mu}) \cdot \hat{n}}{KRc} \right)_{\rm ret} \tag{3}$$

where $K = 1 - \hat{n} \cdot \vec{\beta}$, $\vec{\beta} = \vec{v}/c$ and $\vec{\mu}$ is the magnetic dipole of the pulsar.

Monaghan (1968) also gives an especially useful expression for the actual fields:

$$\begin{aligned}\vec{E} = \Bigg[& \frac{3(\vec{p} \cdot \hat{n})\hat{n} - \vec{p}}{R^3} + \frac{R}{c}\frac{d}{dt}\left(\frac{3(\vec{p} \cdot \hat{n})\hat{n} - \vec{p}}{R^3} \right) \\ & - \frac{d}{dt}\left(\frac{3(\vec{p} \cdot \hat{n})\hat{n} - \vec{p}}{c^2R^2}\frac{dR}{dt} + \frac{\vec{\mu} \wedge \hat{n}}{KcR^2} \right) \\ & - \frac{1}{c^2}\frac{d^2}{dt^2}\left(\frac{\hat{n} \wedge (\hat{n} \wedge \vec{p}) - \vec{\mu} \wedge \hat{n}}{KR} \right) \Bigg]_{\rm ret}\end{aligned} \tag{4}$$

where of course eq. 1 applies. Correspondingly, one also obtains:

$$\begin{aligned}\vec{B} = \Bigg[& \frac{3(\vec{\mu} \cdot \hat{n})\hat{n} - \vec{\mu}}{R^3} + \frac{R}{c}\frac{d}{dt}\left(\frac{3(\vec{\mu} \cdot \hat{n})\hat{n} - \vec{\mu}}{R^3} \right) \\ & - \frac{d}{dt}\left(\frac{3(\vec{\mu} \cdot \hat{n})\hat{n} - \vec{\mu}}{c^2R^2}\frac{dR}{dt} - \frac{\vec{p} \wedge \hat{n}}{KcR^2} \right) \\ & + \frac{1}{c^2}\frac{d^2}{dt^2}\left(\frac{\hat{n} \wedge (\hat{n} \wedge \vec{\mu}) + \vec{p} \wedge \hat{n}}{KR} \right) \Bigg]_{\rm ret}\end{aligned} \tag{5}$$

The reason why these expressions are useful is that all radiation terms are contained in the last term on the rhs of eq. 4 (and 5):

$$\vec{E}_{\rm rad} = -\frac{1}{c^2}\frac{d^2}{dt^2}\left(\frac{\hat{n} \wedge (\hat{n} \wedge \vec{p}) - \vec{\mu} \wedge \hat{n}}{KR} \right)_{\rm ret} \, . \tag{6}$$

Like we said above, we completely neglect the star's rotation, so that $\vec{\mu}$ is a constant and $\vec{p} = \vec{\beta} \wedge \vec{\mu}$. Expanding the above expression to lowest order in β we find:

$$\vec{E}_{\rm rad} \approx \frac{(\hat{n} \cdot \mu)(\vec{\ddot{\beta}} \wedge \hat{n})}{Rc^2} \tag{7}$$

We can now specialize to the case of a binary pulsar, each with magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$, $\hat{n}_1 = \hat{n}_2$, and $\vec{\beta}_1 = -\vec{\beta}_2 = \vec{\beta}$ (for equal mass stars, of course) to obtain

$$\vec{E}_{\rm rad} = \frac{(\hat{n} \cdot (\vec{\mu}_1 - \vec{\mu}_2))(\vec{\ddot{\beta}} \wedge \hat{n})}{Rc^2} \; , \; \vec{B}_{\rm rad} = \hat{n} \wedge \vec{E}_{\rm rad} \, . \tag{8}$$

From Poyinting's vector, $c\vec{E}_{\rm rad} \wedge \vec{B}_{\rm rad}/4\pi$ we obtain the total radiated power as

$$P_{\rm M} = \int d\Omega \frac{(\hat{n} \cdot (\vec{\mu}_1 - \vec{\mu}_2)^2 (\vec{\ddot{\beta}} \wedge \hat{n})^2}{4\pi c^3} \, . \tag{9}$$

We can now, first average over the orbital phase, then carry out the integral over angles (see the Appendix) to obtain:

$$P_{\rm M} = \frac{\omega^4 \beta^2}{15 c^3}\left(3(\mu_1 - \mu_2)_\perp^2 + 4(\mu_1 - \mu_2)_{\rm z}^2 \right) \tag{10}$$

where $\mu_{\rm z}$ the component of the magnetic field along the orbit's normal, while $\mu_\perp$ is the component in the plane of the orbit.

This formula clearly describes the radiation from a magnetic quadrupole, it is a full factor β^2 smaller than the dipole term. It has partially been derived before: Harrison & Tademaru (1975) studied the radiation from a pulsar with a (single) magnetic dipole displaced by a fixed amount from the rotation axis. In their case, they had dipolar radiation because the components of $\vec{\mu}$ in the orbital plane do not remain constant as a consequence of the star rotation. However, the $\mu_{\rm z}$ component produces a quadrupole term which they computed, and coincides with our result above.

We now specialize to the case of a binary pulsar whose orbital decay is driven by gravitational wave radiation losses, in which case (as we are about to see) losses are completely dominated by GW. It can be seen from the equation above that electromagnetic losses depend on the star-to-star distance a as a^{-7}, and are thus strongly peaked around the first moment of physical contact; we may thus

safely assume that the orbit has already been circularized by GWs, and consider only circular orbits. We have

$$\omega^2 = \frac{GM}{a^3} \quad , \quad E = -\frac{1}{2}\frac{G\mathcal{M}M}{a} \tag{11}$$

where the total mass and reduced mass are given, respectively, by $M = M_1 + M_2$, $\mathcal{M} = M_1 M_2/M$; the GW loss rate is

$$\dot{E}_{\rm GW} = \frac{32}{5}\frac{G^4}{c^5}\frac{M^3\mathcal{M}^2}{a^5} \ . \tag{12}$$

From the above we find

$$\frac{da}{dt} = -\frac{2\dot{E}_{\rm GW}a^2}{GM\mathcal{M}} \ . \tag{13}$$

Thanks to the equation above, in the limit $M_1 = M_2$, we can transform eq. 10 into

$$\frac{dE_{\rm M}}{da} = \frac{1}{384}\frac{\mu^2 x}{a^4} \tag{14}$$

where we have called $E_{\rm M}$ the amount of orbital energy lost via electro-magnetic processes ($P_{\rm M} \equiv dE_{\rm M}/dt$) and

$$x \equiv \frac{3(\vec{\mu}_1 - \vec{\mu}_2)^2_\perp + 4(\mu_{1z} - \mu_{2z})^2}{\mu_1^2 + \mu_2^2} \tag{15}$$

and $\mu^2 \equiv \mu_1^2 + \mu_2^2$. Integrating over a we obtain the total amount of radiation emitted by the magnetic quadrupole term:

$$E_{\rm M} = \frac{1}{1152}\frac{\mu^2 x}{a^3_{\rm min}} \ . \tag{16}$$

For typical values $\mu = 10^{30}$ G cm^3, $x \approx 3$ and $a_{\rm min} = 2\times10^6$ cm, we obtain

$$E_{\rm M} \approx 6.5\times10^{38}\left(\frac{\mu}{10^{30}\ \rm G\ cm^3}\right)^2 \ \rm erg \ . \tag{17}$$

This amount of radiation must accompany the final stages of a binary pulsar. It is small compared to the total amount radiated away by GWs, as we anticipated. It is strongly concentrated toward the latest moments in the binary existence, because $P_{\rm M} \propto a^{-7} \propto t^{-7/4}$. The above estimate assumes that fields as large 10^{12} G exist up to the time of merger. However, even a field as low as 10^{10} G may produce a detectable amount of radiation. Thus the above mechanism essentially predicts the existence of radio bursts of roughly millisecond duration.

Though admittedly the estimate above is a text-book exercise, we have been unable to find it in the literature. Its importance lies in the illustration of the fact that the ultimate energy reservoir is the binary's orbital energy, and in the fact that, even under highly idealized assumptions, it leads to a potentially detectable signal.

Still, we show below that another charge-acceleration mechanism exists, which is likely to lead to an estimate of radiated electromagnetic energy dwarfing this one.

3 INDUCTION

In order to see why induction electric fields will be present when both stars have a sizable magnetic field, we consider first two stars *in vacuo*, neglecting the stars' rotation but including their orbital motion. In this case, the side of the field which is brought by orbital motion closer to the companion is compressed by the companion's magnetic field, but, as theorbital motion leads it away from the companion, it is free to expand again. It is this rhythmic compression which generates locally a transient magnetic field, hence an induction electric field.

This periodic effect is illustrated in Fig. 1, where the position of a magnetic line has been computed for identical, point-like magnetic dipoles.

This effect exists also in the opposite limit, when the orbital period is much longer than at least one of the spin periods: again, when a given portion of the magnetosphere is on the day-side it is compressed by the companion's field, while it expands as it moves toward the night-side.

There are two special circumstances under which this effect does not take place. If the stars could synchronize their orbital and rotational motions, thanks for instance to tidal forces, then they would present to their companion always the same portion of their magnetic field, which would not be rhythmically compressed. However, following the work of Bildsten & Cutler (1992), we know that neutron stars are too small for tidal forces to produce orbit-spin synchronization.

Also, the induction field vanishes if the two spin axes and the orbital angular momentum are parallel, as a consequence of the system's reflection symmetry.

We now consider what happens when we allow for the presence of free charges inside the magnetosphere. At this point, one might reason as Goldreich & Julian (1969): given the abundance of free charges, and the fact that they move freely along magnetic field lines, they may redistribute themselves so as to short out the component of the electric field along $\vec{B}$, yielding the transversality condition,

$$\vec{E}\cdot\vec{B} = 0 \ . \tag{18}$$

This hunch is fully borne out by the numerical solution of Contopoulos et al. (1999) in the (stationary!) case of the isolated, rotating aligned pulsar (IRAP for short). In a time-dependent situation, the same spontaneous redistribution of charges will try once again to enforce the transversality condition, eq. 18, and it will succeed to the extent that we may assume charge carriers to be infinitely fast, but we may safely expect this to fail, to some extent, when account is taken of the finite speed of charge carriers.

This situation is similar to the penetration of electromagnetic waves into a good conductor, a textbook problem (Jackson 1975): so long as the speed of the charge carriers is assumed infinite, there can be no penetration of the wave fields inside the conductor, but when account is taken of either the finite speed of the charge carriers, or of the existence of a non-zero, albeit small, resistivity, then the wave fields are able to penetrate for a skin depth inside the conductor.

In our problem, we do not really have to take account of a change of speed for charge carriers due to a finite resistivity: after all, magnetospheric electrons and positrons are likely to be always fully relativistic, hence $1 - v/c \ll 1$. Instead, we must take into account the fact that, as GWs drive the binary into ever closer orbits, the forcing-perturbation time-scale becomes comparable to the time it takes perturbations to propagate inside the magnetosphere, and thus to the time it takes to restore equipotentiality. This way the assumption that charge carriers are moving extremely fast is surely violated.

We may derive the orbital time from eq. 11, using $r = a/2$ as each star's distance from the center of mass of the system, $T_{\rm orb} = 4\sqrt{2}\pi r^{3/2}/(GM)^{1/2}$. For perturbations propagating in the poloidal plane, we may take as the time to re-establish equilibrium the time $T_{\rm cc}$ that charge carriers take to follow (at speed c) a pure magnetic dipole field line; these lines have equation $d = D\sin^2\theta$, where the constant D is the maximum distance of the line from the

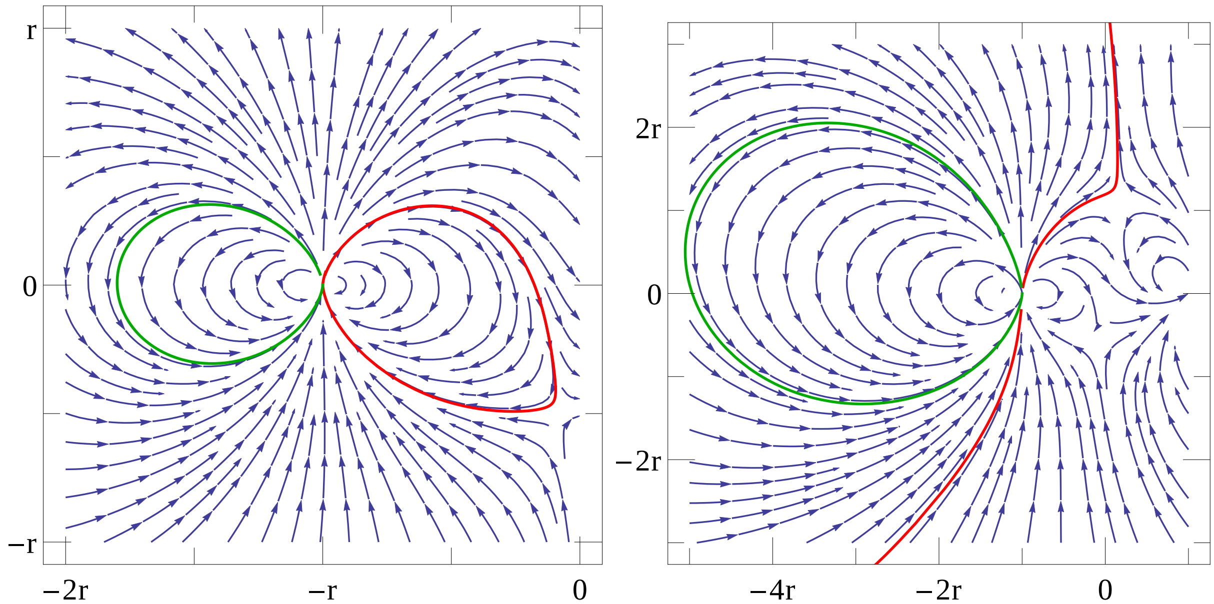

Figure 1. Deformation of field lines from dayside (red) to nightside (green) for two identical, point-like dipoles in $(-r; 0)$ and $(r; 0)$ oriented (respectively) at 90°and 45°; the line in the upper panel, if unperturbed, is closer to the surface of the star.

star, occurring on the equatorial plane $\theta = \pi/2$. Its length is

$$L = \int_0^\pi r(\theta) d\theta = \frac{\pi}{2} D \,, \tag{19}$$

hence $T_{\rm cc}(D) = \pi D/(2c)$. For the mid-distance line, $D = r$, we find:

$$\begin{aligned} \chi \quad &\equiv \quad \frac{T_{\rm cc}(D=r)}{T_{\rm orb}} = \frac{1}{8\sqrt{2}} \left(\frac{GM}{rc^2}\right)^{1/2} = \\ &= \quad 0.057 \left(\frac{M}{2.8 \times M_\odot} \frac{10\ \rm km}{R_{\rm NS}} \frac{R_{\rm NS}}{r}\right)^{1/2} \,. \end{aligned} \tag{20}$$

For perturbations propagating along the toroidal direction, we can directly compare the perturbation speed (which equals the orbital speed $v_{\rm orb}$) to the speed of light, finding

$$\begin{aligned} \chi' = \frac{v_{\rm orb}}{c} \quad &= \quad 0.22 \left(\frac{GM}{rc^2}\right)^{1/2} = \\ &= \quad 0.22 \left(\frac{M}{2.8 \times M_\odot} \frac{10\ \rm km}{R_{\rm NS}} \frac{R_{\rm NS}}{r}\right)^{1/2} \,. \end{aligned} \tag{21}$$

From these equations, we see that at large distances $\chi, \chi' \ll 1$, hence we expect an accurate screening to occur, but as the two stars approach, $\chi \approx 0.1, \chi' \approx 0.2$, which means the screening will be less than perfect. As a comparison, in a metallic onductor $\sigma \approx 5 \times 10^{17}\ s^{-1}$, while, for optical wavelengths, $\nu \approx 5 \times 10^{14}$ Hz, giving $\chi = \nu/\sigma \approx 10^{-3}$.

By analogy with the finite-resistivity conductor mentioned above, this implies that perturbation fields, due to the time-dependent perturbation caused by the companion star, will penetrate for a finite length inside the (otherwise shielded) magnetosphere, without the local charge carriers being able to short out the component of $\vec{E}$ along $\vec{B}$, hence

$$\vec{E} \cdot \vec{B} \neq 0 \,. \tag{22}$$

We can actually push the analogy further to discuss the skin depth. In conducting media with conductivity σ (in planar geometry!), the wavenumber is found to be (Jackson 1975):

$$k \approx 2\pi(1+\imath)\frac{\sqrt{\sigma\nu}}{c} \tag{23}$$

where ν is the impinging wave frequency. In our case, we can take $\nu = 1/T_{\rm orb}$ and $\sigma \approx 1/T_{\rm cc}$, because $1/\sigma$ is the time scale over which a local charge excess spreads itself over the surface of a conductor, i.e., the time scale to restore equipotentiality, which is the same physical interpretation of $T_{\rm cc}$. We obtain for the skin depth

$$\delta = c\,(T_{\rm cc} T_{\rm orb})^{1/2} \,. \tag{24}$$

As a sanity check, we check that for very large separations ($r \to \infty$) shielding is found to be effective, as expected. In this limit, we cannot neglect the stellar rotation rate Ω with respect to the orbital one, ω, because $\Omega \gg \omega$; each star will then have its own corotating magnetosphere out to a distance c/Ω, with typical time $T_{\rm cc}(c/\Omega) = \pi/2\Omega$ (or shorter for inner lines); in this case, $T_{\rm cc}$ does not depend on r, $T_{\rm orb} \propto r^{3/2}$, and the ratio $\delta/r \propto r^{-1/4} \to 0$, showing that (at large separations) the skin depth is exactly that, a thin layer where perturbations can penetrate. It is worth remarking that, in the case of the binary pulsar, $\delta/r \approx 4.8$ for $T_{\rm cc}(c/\Omega)$ for PSR J0737-3039A, and *a fortiori* $\delta/r > 1$ for the slower PSR J0737-3039B.

Conversely, when $r \to R_{\rm NS}$,

$$\frac{\delta}{r} = \frac{cT_{\rm cc}(r)}{r}\chi^{-1/2} = \frac{\pi}{2\chi^{1/2}} = \mathcal{O}(1) \tag{25}$$

which shows that the skin depth is as large as the whole magnetosphere. In this limit, the analysis leading to the expressions for k and δ becomes invalid. In fact, objects of spherical symmetry smaller than about the mean free path for deflection of conduction electrons (i.e., in our case, $cT_{\rm cc}$) are generally considered to be only weakly affected by skin depth effects, in the sense that the spherical metal conductor is not shielded from outside fields, see for instance Petrov (1981); Morokhov et al. (1981). For this reason, and because $\delta \approx r$ at the moment of merger, we feel justified in neglecting skin depth effects in the latest stages of the binary evolution.

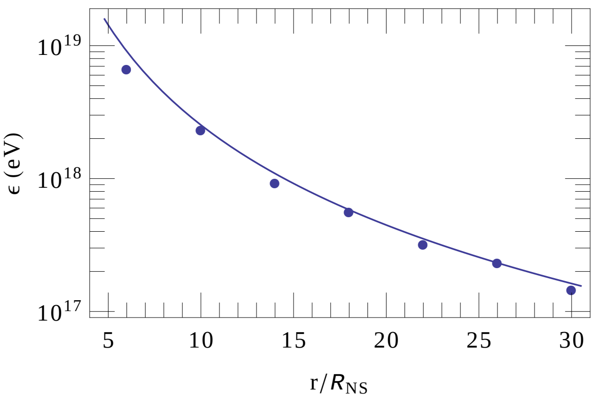

Figure 2. The total electromotive force along a magnetic field line closing at the midpoint between two stars of equal magnetic moments, as the distance between the stars is reduced by GWR. The solid line is an $r^{-5/2}$ fit.

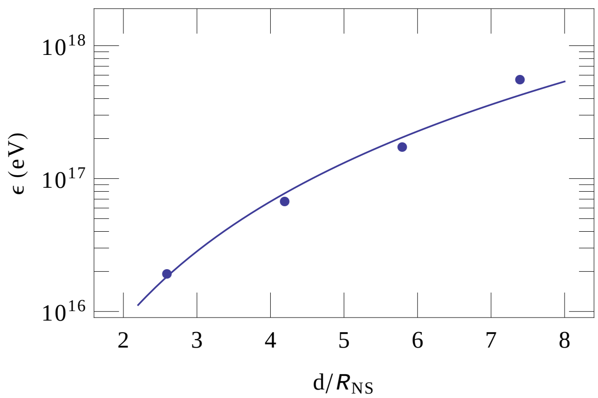

Figure 3. The electromotive force for lines reaching only as far as d, at a given instant. The solid line is a d^3 approximation.

Given the arbitrary orientation of the three axes involved, $\vec{l}, \vec{\mu}_1, \vec{\mu}_2$, and the fact that the skin depth, in the late stages of the binary evolution, is of order of the orbital separation, $\delta \approx r$, it seems difficult to escape the conclusion that, under these circumstances,

$$\vec{E} \cdot \vec{B} \neq 0 \ . \tag{26}$$

3.1 Numerical estimates

We expect emission by particles accelerated by (unscreened) induction electric fields to be strongly concentrated toward the last moments before the binary merges, exactly like the quadrupole radiation we considered above. Hence we shall neglect shielding, as justified above. Also, we are interested in the total electromotive force along closed magnetic field lines, both because we still expect synchrotron losses to constrain particles to follow magnetic field lines, and because the corotating magnetosphere is where the largest particle density is located, and is thus likely to be where most radiation is emitted.

In order to get an idea of the size of this effect, we use as a model the *in vacuo* magnetosphere, using once again Monhagan's expressions, since they represent the *exact* solutions for the fields of point–like magnetic dipole, in the *in vacuo* case. We choose arbitrary orientations of the magnetic dipoles such that $(\vec{\mu}_1 \wedge \vec{\mu}_2) \cdot \vec{l} \neq 0$, where $\vec{l}$ is the orbital angular momentum. A configuration for the magnetic field for a given, generic choice for the magnetic moments is given in fig. 1.

On the line connecting the two dipoles, the two magnetospheres touch approximately where $\mu_1/R_1^2 \approx \mu_2/R_2^3$, with $R_1 + R_2$ the stellar separation. We now compute numerically, for the *in vacuo* case, the integral

$$\mathcal{E} = \int \vec{E} \cdot d\vec{s} \tag{27}$$

for the part (outside the star) of a magnetic field line which passes close to R_1, i.e., roughly the line separating the two magnetospheres. The numerical result for the exact fields given by Monhagan (eqs. 4 and 5) is displayed in fig. 2. Here we have chosen $\mu_1 = \mu_2 = 10^{30}$ in cgs units, from which $R_1 \approx R_2$ (the approximate sign, instead of the equality sign, results from the different inclinations of two dipoles). Please notice that the different dots refer to different field lines: we are comparing the electromotive force always for the line separating the two magnetospheres, which changes as GW reduces the stars' separation $2r$.

Our approximation, $\mathcal{E} \propto r^{-5/2}$ is also displayed in fig. 2. It can be obtained by means of the following argument. We are interested in $\mathcal{E}$ when the stars are very close and in a region very close to *both* stars, in which case we see from eqs. 4 and 5 that the fields are basically given by the formulas for uniform translation: the remaining terms are corrections of order $\omega r/c$. Physically, since each star moves roughly in a straight path over a time scale shorter than v/a, where $a = v^2/r$ is the acceleration, the fields inside a distance cv/a are not affected by time-delay effects, and equal the fields due to a dipole in uniform translation. The condition $r < cv/a$ can easily be rewritten as

$$r < \frac{c}{\omega} \tag{28}$$

i.e., it restricts the domain of validity of the approximation to the volume inside the light cylinder defined in terms of the orbital angular speed, ω. Then the electric field generated by star 1 is

$$\vec{E}_1 \approx \frac{\vec{v}_1}{c} \wedge \vec{B}_1 \ , \tag{29}$$

where $\vec{v}_1$ is the velocity of star 1. This component of the electric field is perpendicular to the magnetic field $\vec{B}_1$ of the star itself. We also add the contribution due to the *other* star (the one we called 2) to the total electric field

$$\vec{E} = \vec{E}_1 + \vec{E}_2 \approx \frac{\vec{v}_1}{c} \wedge \vec{B}_1 + \frac{\vec{v}_2}{c} \wedge \vec{B}_2 \ , \tag{30}$$

which, in general, is not perpendicular to the total magnetic field $\vec{B} = \vec{B}_1 + \vec{B}_2$. The component of the induction electric field along the magnetic field is

$$E_\parallel = \vec{E} \cdot \frac{\vec{B}}{|\vec{B}|} \approx \frac{\vec{v}_1 - \vec{v}_2}{c} \cdot \frac{\vec{B}_1 \wedge \vec{B}_2}{|\vec{B}|} = \frac{\vec{v}}{c} \cdot \left(\hat{n} \wedge \vec{B}_2\right) \ , \tag{31}$$

where $\vec{v} = \vec{v}_1 - \vec{v}_2$ is the stars' relative speed and $\hat{n}$ is the unit vector along the magnetic field $\vec{B}$, in the case of nearly equal–mass stars. Recalling that $B_2 \propto r^{-3}$, $v \propto r^{-1/2}$, and the length over which $\vec{E}$ is to be integrated is $\propto r$, we arrive at the approximation $\mathcal{E} \propto r^{-5/2}$.

The meaning of this approximation is the following. Close to the point where the two magnetospheres have roughly equal values of $B_1 \approx B_2 \approx B$, we find $E_\parallel \approx vB_2/c \approx vB/c$, *i.e.* the parallel electric field is as big as the maximum achievable. Far from this region, *i.e.* closer to the surface of star 1 (where $B_2 \ll B_1 \approx B$), the magnetic line will be nearly exactly parallel to $\vec{B}_1$, and the induction field nearly due to the motion star 1 alone, $\vec{E}_1 \approx$

$\vec{v}_1 \wedge \vec{B}_1/c$: thus $E_\parallel \approx vB_2/c \ll vB/c$, *i.e.* the parallel electric field acts as a small perturbation.

As an order of magnitude, we take for $\vec{E}$ the value estimated above for the *in vacuo* case:

$$\mathcal{E} \approx 7 \times 10^{19} \eta \left(\frac{M}{2.8 \times M_\odot} \right)^{1/2} \left(\frac{B_2}{10^{12}\,\mathrm{G}} \right) \left(\frac{R_{\rm NS}}{r} \right)^{5/2} \mathrm{eV} \; , \tag{32}$$

where $r = R/2$ is the half-distance between the stars, B_2 is the dipolar component of the magnetic field at the surface of star 2, and $\eta \approx 1$ is a fudge factor depending on the relative directions of $\vec{\mu}_1$ and $\vec{\mu}_2$, the choice of the magnetic field line along which we perform the numerical integration, the choice of phase in its travel from dayside to nightside and *vice versa*[1]. This analytical computation is borne out by a numerical integration, in fig. 2, for arbitrary orientation. The scaling with r applies to all orientations we investigated.

The reason why we chose, among all possible magnetic field line, the last one closing onto a star, in the numerical estimate above, is that $\mathcal{E}$ has a maximum for this line. In all other cases, the total magnetic field is dominated by the nearer star, the induction electric field is also dominated by the nearer star, but for these $\vec{E}_1 \cdot \vec{B}_1 = 0$. The only effect is due to the component $E_2 \approx vB_2/c$, which however decreases due to the larger distance from the star 2. This is shown in fig. 3, where $\mathcal{E}$ is computed for fixed orbital separation, but for different distances d from star 1 at which the line crosses the line joining the two stars' centers.

4 CONCLUSIONS

In this paper, we have begun an investigation of the electrodynamics of binary pulsars. In analogy with IRAPs, we have first determined the rate of energy loss due to (quadrupolar, not dipolar) radiation, showing that it is dwarfed by GW losses, but may yet be detectable due its strong transient character, and its near periodicity. We have also discussed the presence of transient induction electric fields, which, *in vacuo*, cause extremely large electromotive forces along magnetic field lines in the corotating magnetosphere. We have also argued, on the basis of an analogy with good conductors, that the skin-depth of these transient fields in a realistic (i.e., charge-rich) magnetosphere, is large once the stars are close to merging, while it is truly thin at large distances from the star. This implies that free charges in the joint magnetosphere will be subject to large electric fields with a component parallel to the total magnetic field. The consequences of this simple observation will be discussed elsewhere.

APPENDIX

Here we derive eq. 10 from eq. 9. We take the axis z to be perpendicular to the plane of the binary star motion, so that, in Cartesian coordinates, $\dot{\vec{\beta}} = \omega^2 \beta (\cos\omega t, \sin\omega t, 0)$. Also, $\vec{\mu}_1 - \vec{\mu}_2 = (\mu_{\rm x}, \mu_{\rm y}, \mu_{\rm z})$, and $\hat{n} = (\cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi)$. We thus find

$$\hat{n} \cdot (\vec{\mu}_1 - \vec{\mu}_2) = \mu_{\rm x} \cos\theta + \mu_{\rm y} \sin\theta \cos\phi + \mu_{\rm z} \sin\theta \sin\phi \; , \tag{33}$$

and

$$(\dot{\vec{\beta}} \wedge \hat{n})^2 = \omega^4 \beta^2 \left[1 - (\cos\omega t \cos\theta + \sin\omega t \sin\theta \cos\phi)^2 \right] \; . \tag{34}$$

The time average over the orbital period is now trivial, and we obtain:

$$\begin{aligned} P &= \frac{\omega^4 \beta^2}{4\pi c^3} \int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi \times \\ & \quad (\mu_{\rm x} \cos\theta + \mu_{\rm y} \sin\theta \cos\phi + \mu_{\rm z} \sin\theta \sin\phi)^2 \times \\ & \quad (1 - \frac{1}{2} \cos^2\theta - \frac{1}{2} \sin^2\theta \sin^2\phi) = \\ &= \frac{\omega^4 \beta^2}{15 c^3} (3\mu_\perp^2 + 4\mu_{\rm z}^2) \end{aligned} \tag{35}$$

[1] Strictly speaking, there are also relative orientations of $\vec{\mu}_1$ and $\vec{\mu}_2$ for which the above integral necessarily vanishes. However, this happens only when both the magnetic moments are strictly parallel to $\vec{\omega}$, a sufficiently rare occurrence that we may neglect in the following.

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